A Problem in General Relativity

by Edward Cree

Consider a Schwarzschild black hole of mass M. The radius of the event horizon will be

$r_s=2\mathrm{GM}/c^2$.

However, the gravitational potential at a distance r from the singularity will be

$V=-\mathrm{GM}/r$.

There will be a sphere of points at a distance r such that $V+c^2=0$.
At such a point,

$-\mathrm{GM}/r=-c^2$

hence

$r=\mathrm{GM}/c^2$

thus

$r=r_s/2$.

Hence, at all points within this sphere, the mass-energy of any matter will be at least cancelled out by its (negative) gravitational potential.
That is, for a mass m,

$mc^2-\mathrm{GM}m/r\leq 0$

Which being the case, there is no energy required for the creation of matter on the surface of the sphere – in fact, within the sphere, energy would be released – and hence, in theory at least, matter should spontaneously be created; the energy required to make it being 'locked up' in the negative gravitational potential energy.
Furthermore, the spontaneous creation of matter will increase the mass of the black hole, thus increasing the radius r_s, and hence, the radius r of the mass-creation region. This would increase the volume of that region, and thus the rate at which matter would be created.
This result would suggest that black holes grow exponentially in mass. This is, however, in contradiction with observations that black holes only gain mass at the rate at which it falls into them from outside.
To calculate further, let us first assume that, for a volume V within the mass-creation region,

$\mathrm{dM}/\mathrm{d}t \ \alpha \ V$

Then $\mathrm{dM}/\mathrm{d}t \ \alpha \ (4\prod/3)r^3$
hence $\mathrm{dM}/\mathrm{d}t = \mathrm{k}r^3$
where k is a constant of proportionality.
However, $r=r_s/2 = \mathrm{GM}/c^2$
hence $\mathrm{dM}/\mathrm{d}t = \mathrm{kM}^3$ (where k is a different k to before)
thus $\mathrm{M}^{-3}\mathrm{dM}/\mathrm{d}t = \mathrm{k}$
hence $-\mathrm{M}^{-2}/2 = \mathrm{kt} + \mathrm{c}$ (where c is a constant of integration)
Thus, within a finite time, M will tend to infinity. This means that, within finite time, the universe will be engulfed by an infinite black hole – which cannot possibly be allowed to occur as this would cause an entropy problem: the entropy of a black hole of a given mass is minimal, due to the no hair result. But at the heat death of the universe, i.e. a time t = infinity, entropy must be maximal, since entropy always increases with passage of time. Thus, even if the rate of mass creation were sufficiently slow that it would not be directly observed, the obvious absurdity of unlimited spontaneous mass growth would lead to contradictions of other physical laws.

It is clear that this phenomenon does not, in fact occur. However, General Relativity appears to suggest that it should. The implication of this is that there may be a problem in General Relativity – hence the title.

Although it is to be expected that this problem has been considered, and furthermore solved, already – and may even be viewed as trivial – the author would, nonetheless, wish to hear of the nature of such a solution.