

## **A Problem in Special Relativity**

*by Edward Cree*

Consider two bodies (A and B) travelling away from Earth (E) in opposite directions, with velocities (relative to Earth) of, say,  $v\mathbf{i}$  and  $-v\mathbf{i}$ , where  $\mathbf{i}$  is a unit vector, and  $v > (c/2)$ .

To an observer on A, the velocity of B will still appear to be less than  $c$ , due to relativistic effects; the velocity of B in A's frame of reference is  $2v/(1+v^2/c^2)$ , so even if  $v=c$ ,  $v_{B/A}=c$ .

However, to an observer on Earth, the separation speed is  $s_s=2v$ , hence  $s_s > c$ . If body A was absolutely at rest, since body B could not be travelling faster than  $c$ ,  $s_s$  could not exceed  $c$ .

Therefore, as  $s_s > c$ , A cannot be absolutely at rest.

If  $v$  is, in fact, the limit as  $v$  tends to  $c$ , then an observer on Earth could deduce that, since A is receding at  $c$ , the observer cannot be (absolutely) travelling in the direction of A, else A would be exceeding  $c$ . Similarly, the observer cannot be (absolutely) travelling in the direction of B, else B would be exceeding  $c$ . If this situation were replicated on the  $\mathbf{j}$ - and  $\mathbf{k}$ -axes, it could be deduced that E is absolutely at rest. This is, however, a contradiction of Special Relativity, which states that absolute speed cannot be determined!

To consider the problem further, allow  $v$  again to take any value such that  $(c/2) < v < c$ , and consider now that a mirror M is attached to A such that if the observer on Earth looks at the mirror, they would expect to see B, which they can of course see in the normal way by looking at B. However, in the mirror, the image of B is not visible, since the light from B can never (in E's frame of reference) reach M. Thus the observer can deduce that A is not absolutely at rest (since  $s_s > c$ , as before). This, again, contravenes Special Relativity.

Although it is to be expected that this problem has been considered, and furthermore solved, already – and may even be viewed as trivial – the author would, nonetheless, wish to hear of the nature of such a solution.