



Hybrid effects on vehicle stability

It's now been a couple of years since the rocketry world started launching with impunity kits designed for solid propulsion, but with a hybrid motor inserted up them instead.

So far, there have been surprisingly few reported problems.

Or rather, it hasn't been realised that the new crop of weird things that occasionally happen may have been caused by the hybrid.

So what are the extra effects to take note of?

Words in **bold** appear in the Glossary at the end of the document.

Centre of Gravity shift

First off, there's the fairly obvious point that the Centre of Gravity (CG) of a rocket vehicle powered by a solid motor moves nosewards during the burn as it ejects propellant mass that was in a motor that typically was aft of the CG.

The vehicle ends up a lot more stable at burnout than it was at launch.

The vehicle's **Barrowman** static stability (in **Calibers**) has the most noticeable effect on the vehicle's trajectory where the vehicle is travelling at low speed: launch and apogee.

Stability at launch is the one we all know about, and we tune the **Calibers** accordingly.

At apogee, the vehicle is much more stable, so there's no need to bother worrying about stability there.

Depending on the particular hybrid system, the vehicle CG can actually track tailwards during the burn, because there's all that oxidiser mass pouring out of a tank that is ahead of the CG. In moderation (i.e provided it doesn't go unstable), this is a useful effect, because the ensuing trajectory remains more vertical: The length of the moment arm between CG and Centre of Pressure (CP) is smaller, providing gravity with less torque to turn the trajectory over.

But the stability at apogee is a lot less than you'd get with a solid motor.

On the first flight of Aspire's FLARE hybrid vehicle, it was hit by a windshear just before apogee as it cleared the wind-shadow of a nearby ridge.

It tumbled, reducing apogee height, and this could have affected drogue deployment if there had been a drogue deployed at apogee.

Aspire now sim the CG over the whole burn to check that it doesn't dip too low at any point, and increase the launch static margin if we have to. This does require a long launchrail to keep the trajectory straight just after launch, but this is a small price to pay for a clean flight.

To estimate the CG shift during the burn, assume that the flowrate of nitrous out of the tank (and therefore the speed at which the surface of the liquid is falling in the tank) is directly proportional to the instantaneous thrust.

This reflects the fact that both the thrust and oxidiser flowrate fall as the tank pressure falls during the burn, and for nitrous at least, reflects that fact that the performance is fairly insensitive to the changing mixture ratio of fuel to oxidiser during the burn.

So a graph of the speed that the level in the tank is dropping versus time is pretty much the same shape as the thrustcurve.

Scale this level-dropping speed so that the tank just empties at burnout.

If you have accurate mass flowrate data for the nitrous, then do use that in preference, but bear in mind that some of the nitrous at the surface vaporises as the tank empties so the level falls faster than the mass continuity equation would predict.

(see our paper on nitrous oxide for details.)

Integrate this level-dropping speed with time to get the level of the fluid with time during the burn. (e.g. new level = previous level - speed times dt, where dt is, say, a quarter-second time interval.)



The mass of the fluid is then its density times its volume (= tank cross-sectional area times fluid height), and the fluid's own CG is then in the middle of the column of fluid, (i.e. at half the height of the fluid.)

Remember that the nitrous vapour in the tank is almost as dense as the liquid nitrous below it on a hot day; you can't ignore the mass of the vapour.

It occupies the entire space above the level of the liquid, and again, it's own CG is pretty much in the middle. (i.e. halfway down from the top of the tank to the height of the fluid surface.)

Large Polar Moment of Inertia

The polar axes are the pitch and yaw axes, not the longitudinal (roll) axis.

The long, heavy tanks supplied with commercial hybrid systems give the vehicle a high Moment of inertia about its pitch/roll axes.

This can make hybrid vehicles prone to **Roll resonance**, because the weathercock frequency reduces as the polar Moment of Inertia increases, low enough to hit the roll rate. (see glossary.)

Aspire's early FLARE rocket utilised a particularly heavy paintball tank.

The 2nd FLARE flight spun-up in roll due to an aerodynamic fairing that wasn't aligned properly, and hit roll resonance just after launch.

FLARE eventually gained enough airspeed to leave resonance: it stopped corkscrewing, but the trajectory had been skewed from the near-vertical launch into a long, low path with the usual excessive apogee airspeed that broke the recovery system.

FLARE was ignominiously dug-out with a spade.

...of the liquid

Here's food for thought: what's the polar moment of inertia of a liquid? (the moment of inertia of the nitrous about the *roll* axis is effectively zero.)

This is a really thorny little problem that has occupied some of science's best minds over the centuries, and as the mass of oxidiser in a good hybrid system should be a large part of the launch mass, it strongly affects the dynamic stability, i.e. the response of the vehicle to gusts. Some trajectory sims that model these dynamic effects require you to input the moment of inertias of the vehicle's components; what's the correct value for the liquid within the hybrid's oxidiser tank?

The moment of inertias of the parts of the vehicle that are solid are easily calculated using standard equations, but the nitrous clearly isn't solid.

Observation of different shapes of clear-plastic bottles of water being rotated by hand shows the main effects.

Far from the axis of rotation, the fluid is carried round by the walls of the bottle as the bottle rotates, so the moment of inertia here is almost as much as if the fluid were frozen in the bottle, i.e as if it were solid.

The fluid near the axis of rotation of the bottle however, a roughly spherical region of the same diameter as the bottle, isn't affected by the bottle's rotation.

As the bottle rotates, this roughly spherical shape doesn't rotate, as if it were a free-to-rotate solid sphere on a low-friction pivot.

So this central region effectively has zero moment of inertia.

Between these two regions, the effects merge from one to the other: partial rotation.

If the tank is long and thin then the tank diameter is small in comparison to the scale of the tank, so the central non-rotating region of fluid is small; the tank's moment of inertia is almost as large as if the fluid was completely frozen solid.



But if the tank is squat, OR the axis of rotation of the tank is near the base of the tank and the tank is almost empty, then most of the fluid is not rotating, so the moment of inertia is very much lower than the value it would be if the fluid were frozen solid.

Engineer's mathematical models of liquid tanks use a simple approximation of the above picture.

Reference 1 is typical, where the fluid is replaced by a solid cylinder (as if it were frozen solid) but with a free-to-pivot disk or sphere at the axis of rotation that is sized to represent the loss of rotational inertia in this region.

Reference 1's mathematical derivation of the exact values of the frozen and rotating parts of the model involves some powerful maths.

The fluid is modelled as frictionless and incompressible, and mathematically irrotational (a mathematical construct that says that though successive lumps of fluid rotate as they slide around each other, the lumps themselves do not rotate, rather like the cars on a fairground Ferris wheel.)

These assumptions allow the velocity of the fluid at any point within the tank to be obtained as the tank is rotated.

(For those who are interested, the usual fluid dynamics method of obtaining the velocity potential within the tank using Laplace's equation is performed, noting the boundary conditions at the tank walls and the fluid free surface.)

Having got the velocity at all points within the tank mapped out, Bernoulli's equation is then used to get the pressure distribution around the tank, and then this pressure distribution is integrated over the area of the tank walls and base to get the forces that the fluid exerts on the walls as the tank is rotated.

It takes about 60 pages of equations to perform this integration however, even though a lot of linearization of terms goes on!

The author throws in series expansions of the higher derivatives of Bessel functions before he's satisfied..

Having got the forces that the fluid exerts on the tank walls, Newton's laws are used to figure out what size of fake masses and moments of inertias would cause these actual forces if the masses and inertias were faked as being a mix of solid cylinders and rotating disks as described above.

It's a mathematical tour-de-force, but the results tally with other's analyses as used in the aerospace industry, such as those quoted in references 2 and 3

The resulting equations are:

The moment of inertia of frozen solid fluid in a cylindrical tank about the centre of mass (CG) of the fluid is:

$$I_{frozen} = ma^2 \left[\frac{1}{12} \left(\frac{h}{a} \right)^2 + \frac{1}{4} \right]$$

where: h = the fluid depth, a = the tank internal radius, m = the total fluid mass

From this we *subtract* the inertia of a frozen disk of fluid that is free to pivot about the centre of mass of the fluid: $I = I_{frozen} - I_{disk}$

$$\text{Where } I_{disk} = 8ma^2 \sum_{n=1}^4 \left[\frac{1 - \frac{2}{K} \tanh\left(\frac{K}{2}\right)}{\epsilon_n^2 (\epsilon_n^2 - 1)} \right]$$



Constants for the first four terms come from derivatives of Bessel functions, and are:
 $\epsilon_1 = 1.8412$, $\epsilon_2 = 5.3314$, $\epsilon_3 = 8.5363$, $\epsilon_4 = 11.7060$

and: $K = \epsilon_n \left(\frac{h}{a} \right)$

Strictly, the summation (addition) is to infinite terms $\sum_{n=1}^{\infty}$ but adding together only the first four terms will do in practice as each successive term gets smaller and smaller.

tanh is the hyperbolic tangent function, as found on scientific calculators and most spreadsheet packages.

These results are for a tank with flat end-faces, but are good enough for tanks with domed ends provided that you calculate *h* as if the tank were indeed flat-ended but of the same internal volume as your dome-ended tank.

Use kilograms and metres to get the moments of inertia in kg m^2 and note that the final moment of inertia is about a point that is the centre of mass (CG) of the fluid, i.e. at half the depth of the fluid.

Just like any other moment of inertia, this can be converted to the value about the vehicle CG using the Parallel Axes theorem:

$I_{CG} = I - m d^2$ where *d* in this case is the distance between the centre of mass of the fluid and the vehicle CG, and *m* here is the total mass of the fluid.

Note that though the nitrous vapour in the tank is almost as dense as the liquid nitrous below it on a hot day, it is a vapour and not a liquid, so I've assumed that it has zero moment of inertia.

This may well be too much of an assumption, comments please.

As for the moment of inertia of the rest of the hybrid system including the empty tank, contact the manufacturer, or measure it.

Slosh

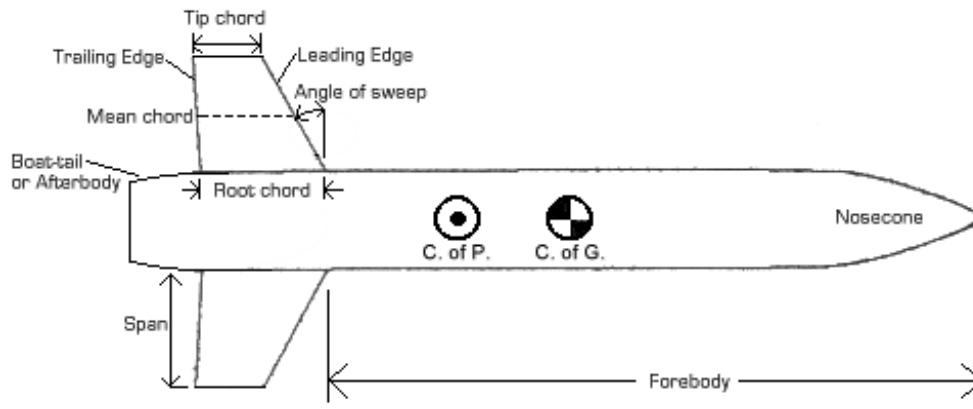
The above analysis of the moment of inertia of the fluid in the tank doesn't care whether the tank is completely full or not: whether the tank has a lid on it doesn't affect the result.

But if the tank has no lid, or if it isn't completely full, then waves will slosh around on the surface of the fluid as the vehicle moves about.

The weight (mass) of these waves hitting the sides of the tank causes lateral forces on the vehicle that upset the trajectory.

Fortunately, the mass of the waves compared to the mass of the whole vehicle depends on the tank internal diameter; for vehicles a lot smaller than those that could launch a satellite, the effects of slosh can be ignored.

For example, Aspire's ADV2b 4-inch diameter vehicle uses a tank of similar diameter to the larger Hypertech tanks. From reference 2 I've calculated that the masses of the sloshing waves are just over one percent of the vehicle's lift-off mass.

Glossary:**Geometric definitions:****Barrowman:**

The basic stability of fixed-fin rocket vehicles has been covered by James Barrowman and others, and won't be repeated here.

(Note that several books and web-pages contains errors in their reprinting of Barrowman's stability equations; it's better to get a copy of the original paper.)

Barrowman's method is a classic static-stability analysis: it simply tells you whether your fins are large enough so that your vehicle has a tendency to keep the nose pointing in the direction of flight as required, and it assumes that the ensuing rotation of the vehicle about its CG is slow enough not to affect the analysis.

Calibers, Calibres:

In rocketry, vehicle dimensions are usually divided by (compared to) the diameter of the thickest part of the fuselage so that rockets of different size can be compared: this diameter is therefore one Caliber.

Centre of Pressure (CP):

The point on the rocket's surface where the average of all the aerodynamic forces from the nose, body, and fins act. This must be behind the Centre of Gravity (CG) by at least one Caliber.

Roll resonance:

Fin-stabilised rockets have a frequency at which they will nod back and forth in pitch in response to, say, a gust of wind. This 'weathercock' frequency changes with airspeed.

If the fins are misaligned and cause the vehicle to spin along its (roll) long axis, then the spin angular frequency (roll rate times 2 pi) might briefly equal the weathercock frequency at some point in the flight, usually at the low airspeeds just after launch, and if the **Barrowman** stability is too low. This causes a corkscrewing trajectory that may later suddenly stabilise pointed in any direction.

On military test-ranges it's been found that liquid mass onboard a rapidly spinning finned projectile makes the vehicle much less stable, so nitrous compounds the problem. So don't misalign the fins: spin stabilisation was a '60's fashion that won't pay off.



References

Ref. 1: 'Fluid oscillations in the containers of a space vehicle and their influence upon stability'
by Helmut F. Bauer

NASA technical report TR R-187

Feb. 1964

Ref. 2: TRW Space Data book, 4th edition

Figure 6.A4, sloshing data

TRW Space and technology group

Ref. 3: 'Wind effects on launch vehicles'

Section 4-4-3 propellant dynamics

The advisory group for research and development, NATO

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